Atom in a coherently controlled squeezed vacuum

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A broadband squeezed vacuum photon field is characterized by a complex squeezing function. We show that by controlling the wavelength dependence of its phase it is possible to change the dynamics of the atomic polarization interacting with the squeezed vacuum. Such a phase modulation effectively produces a finite range temporal interaction kernel between the two quadratures of the atomic polarization yielding the change in the decay rates as well as the appearance of additional oscillation frequencies. We show that decay rates slower than the spontaneous decay rate can be achieved even for a squeezed bath in the classic regime. For linear and quadratic phase modulations the power spectrum of the scattered light exhibits narrowing of the central peak due to the modified decay rates. For strong phase modulations side lobes appear symmetrically around the central peak reflecting additional oscillation frequencies.

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The effect of the interaction of an atom with a squeezed light had been studied extensively. Gardiner [1] has considered the behavior of a two-level atom damped by an infinite bandwidth squeezed vacuum. He showed that the two quadratures of the atomic polarization decay at different rates. Decay rates smaller than the decay rates of spontaneous emission can be achieved for non-classical squeezing. The presence of the two decay rates modify the fluorescence spectrum, Ref. [2]. In Refs. [3, 4, 5] the interaction of a finite bandwidth squeezed vacuum with a two level atom was investigated. Review of these and related studies is found in Ref. [6]. Recently these works were extended in Ref. [7] to include interactions with semiconductor microstructures.

In the field of quantum coherent control pulse shapers [8] were used to attain prescribed phase modulation of a down converted light in order to control two photon absorption. It was shown [9] that pulses can be shaped in a way that will stretch them temporally affecting the transition probability [10]. Experiments [11, 12] have also been performed on two photon absorption with coherent, narrow band down-converted light, demonstrating nonclassical features which appear at very low powers [13, 14, 15] and result from time and energy correlations (entanglement) between the down-converted photon pairs.

In this work we wish to investigate the effect of controlling and modulating the relative phase of the modes of a squeezed reservoir interacting with an atom. This can be easily done by a pulse shaper arrangement as shown in Refs. [9, 16]. We will demonstrate that by controlling in this manner the phases of the correlations in the squeezed reservoir it is possible to control the dynamics of atomic polarization and in particular to further reduce its decay rates.

We will use the standard model to describe the interaction of a two-level atom with a broadband radiation field. The atom is assumed to be coupled to a one dimensional set of radiation modes. The Hamiltonian is given in the electric-dipole and rotating-wave approximations by $H = H_0 + H_I$ as (we take $\hbar = 1$):

$$H_0 = \omega_a \sigma_z + \sum_q \omega_q b_q^\dagger b_q$$
$$H_I = \Gamma\sigma_+ + \sigma_-\Gamma^\dagger \ , \ \Gamma = \sum_q g_q b_q. \tag{1}$$

The pseudospins $\sigma_+, \sigma_-$and σ_z describe the atom, ω_a is the atomic resonance frequency, b_q and $b_q^\dagger$ describe radiation modes with wave vector q and frequency ω_q and g_q are the mode-atom couplings.

We assume that the radiation acts as a reservoir with correlations of a two mode broadband squeezed vacuum [1, 3]

$$\begin{aligned} \langle b_q^\dagger b_{q'}\rangle &= N(\omega_q)\delta_{q,q'} \\ \langle b_q^\dagger b_{q'}^\dagger\rangle &= M(\omega_q)\delta_{2q_0-q,q'} \\ \langle b_q\rangle &= \langle b_q^\dagger\rangle = 0, \end{aligned} \tag{2}$$

where $N(\omega_q)$ describes the average occupations of photonic modes while the magnitude $|M(\omega_q)|$ gives the squeezing strength of mode pairs centered around the frequency $\omega_0 \equiv \omega(q_0)$. The phase of $M(\omega_q)$ describes the "direction" of squeezing in the phase space of squeezed mode pairs. In the common method of generating squeezed vacuum radiation by non-linear down conversion the phase of $M(\omega_q)$ is constant over the frequency range. Letting the radiation pass through a pulse shaper makes it possible to change this phase into a prescribed function of ω_q, cf., Refs. [9, 16].

We shall for simplicity assume that N, $|M|, g_q$ are constants within a bandwidth $\omega_0 \pm B/2$ and that B is $\ll \omega_0$ but is much larger than any other frequency in the system.

Using the equations of motion

$$
\begin{aligned}
\dot{\sigma}_+ &= -i[\sigma_+, H] = i\omega_a\sigma_+ - 2i\Gamma^\dagger\sigma_z \\
\dot{\sigma}_z &= -i[\sigma_z, H] = -i\Gamma\sigma_+ + i\Gamma^\dagger\sigma_- \\
\dot{b}_q &= -i[b_q, H] = -i\omega_q b_q - ig_q^*\sigma_-
\end{aligned} \tag{3}
$$

and transforming them to the rotating frame of the laser frequency, we integrate over time the equations for σ_z and b_q and substitute the result back into the equation for σ_+. We then average the resulting equation over the initial state and assume that the bath and the atom are (approximately) decorrelated, i.e. that the photon atom correlators factorize at all times e.g. $\langle b_q(t)\sigma_+(t')\rangle \approx \langle b_q(t)\rangle\langle\sigma_+(t')\rangle$, etc. The decorrelation assumption is valid in the regime of weak coupling between the system and the photon bath. Using it we obtain a closed equation for the atomic polarizations $\langle\sigma_\pm\rangle$

$$
\begin{aligned}
\frac{d}{dt}\langle\sigma_+\rangle &= \left(i(\omega_a-\omega_0) - \frac{\gamma}{2}\right)\langle\sigma_+\rangle \\
&- 2\int_0^t dt'\{\sum_q |g_q|^2 N(\omega_q) e^{i(\omega_q-\omega_0)(t-t')}\langle\sigma_+(t')\rangle \\
&- g_q^* g_Q^* M(\omega_q) e^{i(\omega_q-\omega_0)(t-t')}\langle\sigma_-(t')\rangle\},
\end{aligned} \tag{4}
$$

where $\omega_Q = 2\omega_0 - \omega_q$, $\gamma = \rho(\omega)|g_q|^2$ is the vacuum atomic decay rate and $\rho(\omega)$ is the density of the radiation modes. We assume that $\rho(\omega)$ is flat over the bandwidth B.

We now transform the sums over q into integrals. We define the following parameters

$$
\begin{aligned}
\gamma\mathcal{N} &= \rho(\omega)|g_q|^2 N(\omega_q) \\
\gamma\mathcal{M} &= \rho(\omega) g_q^* g_Q^* |M(\omega_q)|,
\end{aligned} \tag{5}
$$

and the following function

$$
k(t-t') = \frac{1}{\pi}\int_{-B/2}^{B/2} d\omega e^{if(\omega)} e^{i\omega(t-t')}. \tag{6}
$$

where $f(\omega)$ is the phase of $M(\omega) = |M|e^{if(\omega-\omega_0)}$. Note that $f(\omega) = f(-\omega)$. Without loss of generality we can take $f(0) = 0$ using the freedom to absorb its non-zero value in the phase of $\mathcal{M}$.

With this notation the equation for the atomic polarizations becomes

$$
\frac{d}{dt}\langle\sigma_+\rangle = \left(i\delta - \gamma(\mathcal{N}+\frac{1}{2})\right)\langle\sigma_+\rangle + \gamma\mathcal{M}\int_0^t dt' k(t-t')\langle\sigma_-(t')\rangle, \tag{7}
$$

where we defined the detuning $\delta = \omega_a - \omega_0$.

The phase modulation of the squeezing parameters leads to a finite range memory kernel in Eq. (7) which couples atomic polarizations to their complex conjugates at earlier times. This non-Markovian dynamics is still linear due to the atom-bath decoupling assumption. One can envisage situations in which the non-Markoian dynamics may lead to the breakdown of this assumption but we will not consider such cases here.

To analyze the controlled memory effects we apply the Laplace transform to Eq. (7) and obtain

$$
\begin{pmatrix} \langle\tilde{\sigma}_-(s)\rangle \\ \langle\tilde{\sigma}_+(s)\rangle \end{pmatrix} = \frac{\begin{pmatrix} s + i\delta + \gamma(\mathcal{N}+1/2) & \gamma\mathcal{M}\tilde{k}(s) \\ \gamma\mathcal{M}^*\tilde{k}^*(s^*) & s - i\delta + \gamma(\mathcal{N}+1/2) \end{pmatrix}\begin{pmatrix} \langle\sigma_-(0)\rangle \\ \langle\sigma_+(0)\rangle \end{pmatrix}}{(s+\gamma(\mathcal{N}+1/2))^2 + \delta^2 - |\mathcal{M}|^2\tilde{k}(s)\tilde{k}^*(s^*)} \tag{8}
$$

where we assumed non-zero initial conditions at $t = 0$ for $\langle\sigma_\pm(t)\rangle$ and denoted $\langle\tilde{\sigma}_\pm(s)\rangle, \tilde{k}(s)$ the Laplace transform of the polarizations and the kernel function respectively. The non-zero $\langle\sigma_\pm(0)\rangle$ is the easiest situation to analyze although it can only be realized with a specially designed initial pulse applied before letting the squeezed reservoir interact with the atom. We will later discuss the implications for the fluorescence spectrum. In the following we will assume for simplicity the exact resonance case $\delta = 0$.

The poles of the Laplace transform are the solutions of

$$
s/\gamma = -\mathcal{N} - 1/2 \pm |\mathcal{M}|\sqrt{\tilde{k}(s)\,\tilde{k}^*(s^*)}. \tag{9}
$$

For the unmodulated phase $f(\omega) = 0$ we obtain the usual Markov result $k(t) = \delta(t)$, $\tilde{k}(s) = 1$, in which case $s_\pm = \gamma(\mathcal{N}+1/2\pm\gamma|\mathcal{M}|)$, representing the splitting of the decay rate into the fast and the slow components under the influence of the squeezing, Ref. [1]. For the modulated phase it is convenient to consider the graphic solution, cf. Fig.1. The symmetry of $f(\omega)$ implies that $\tilde{k}(0) = 1$. Therefore for analytic $\tilde{k}(s)$ to the first order in s, $\tilde{k}(s) \approx 1 - s\int_0^\infty tk(t)dt \equiv 1 - sk_1$. The right hand side of Eq.(9) reduces to $-\mathcal{N} - 1/2 \pm |\mathcal{M}|(1 - s\mathrm{Re}k_1)$ and if $\mathrm{Re}k_1 > 0$ then the slow component of the decay rate

$$
s_- = -\gamma(1 - \gamma|\mathcal{M}|\mathrm{Re}k_1)(\mathcal{N} + 1/2 - |\mathcal{M}|) + \ldots \tag{10}
$$

can become even slower than in the Markov case.

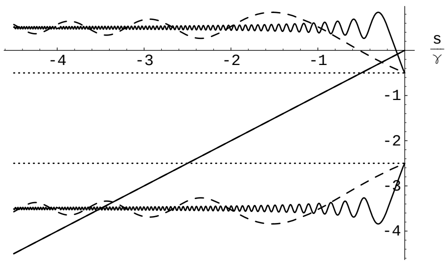

FIG. 1: Graphical solution of Eq. (9) with $\mathcal{N} = \mathcal{M} = 1$ for the quadratic phase modulation, Eq. (12). The wavy lines show the right hand side of the equation for $T\gamma = 5$ (solid line) and $T\gamma = 1$ (dashed line) while the horizontal dotted lines are for $T = 0$ - the standard Markov case.

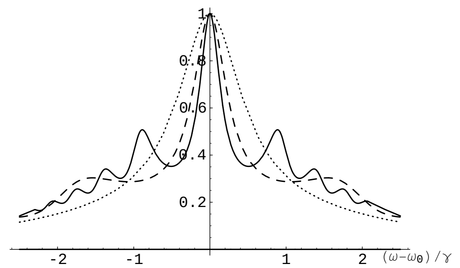

FIG. 2: The fluorescence spectrum for the quadratic phase modulation, Eq. (11) and $\mathcal{N} = \mathcal{M} = 1$. Solid, dashed and dotted lines are for $T\gamma = 5$, $T\gamma = 1$ and $T = 0$ (the Markov case) respectively.

We wish to remark that in addition to the real poles, the complex poles of Eq. (8) in the Re$s < 0$ half plane also play a role in determining the dynamics of the atomic polarizations. Their imaginary parts represent additional oscillation frequencies. If their real part is smaller than the real poles they may dominate the long time decay. Finally possible cuts may also make a contribution. We will discuss such contributions in the examples below.

We consider two concrete examples of the phase modulation. The simplest case to calculate is the *quadratic phase*

$$f(\omega) = T^2\omega^2, \tag{11}$$

where T is a real positive parameter. The resulting Laplace transformed memory kernel in the $B \to \infty$ limit is

$$\tilde{k}(s) = e^{-iT^2s^2}\mathrm{erfc}(\sqrt{-i}Ts). \tag{12}$$

The solution of (9) with this $\tilde{k}(s)$ is shown graphically by plotting the two sides of this equation in Fig.1. One sees the increase of the fast and the decrease of the slow decay rates caused by the phase modulation. In this example $\gamma\mathrm{Re}k_1$ in (10) is $\sqrt{\frac{2}{\pi}}T\gamma$.

For $\tilde{k}(s)$ given by Eq. (12) the structure of Eq. (8) is much richer than that implied by the real poles. One finds in addition numerous complex poles, the positions of which depend on the parameters $\mathcal{N}$, $\mathcal{M}$, T and γ. Their influence can be seen in Fig. 2 where we plot $\langle\tilde{\sigma}_-(s = i\omega)\rangle/\langle\tilde{\sigma}_-(s = i\omega_0)\rangle$. Together with the significant narrowing of the central part of the peak relative to the Markov case $T = 0$ one observes the appearance of the side lobes which reflect a complicated pole structure in the $Res < 0$ half plane.

The quantity plotted in Fig. 2 can be related to the fluorescence spectrum of light emitted by the atom into empty modes of the radiation field. This spectrum is given by

$$S(\omega) = \frac{\gamma}{2\pi}\mathrm{Re}\Big\{\int_0^\infty \mathrm{d}\tau\, e^{-i\,\omega\tau}\langle\sigma_+(t_0+\tau)\sigma_-(t_0)\rangle_{ss}\Big\}. \tag{13}$$

where the subscript ss denotes steady state. This expression involves the average of the two time product of polarization operators $\langle\sigma_+(t_0+\tau)\sigma_-(t_0)\rangle$. It is not difficult to show that for $t_0 = 0$ and $t_0+\tau = t$ this average satisfies the same dynamical equation (7) as a single time average $\langle\sigma_+(t)\rangle$ provided that one uses the atom - bath decorrelation assumption. Irrespectively of the initial condition the system relaxes to a steady state regime. One can therefore replace the initial time, i.e. the low limit $t = 0$ in the integral in Eq. (7) by t_0 provided it is chosen after the steady state is reached. The initial condition is then given by $\langle\sigma_+(t_0)\sigma_-(t_0)\rangle_{ss} = 1/2 + \langle\sigma_z(t_0)\rangle_{ss}$. The Laplace transformed solution of this set is given by (8) with $\langle\widetilde{\sigma_\pm\sigma_-}\rangle(s)$ replacing $\tilde{\sigma}_\pm(s)$ in the left hand side and $\langle\sigma_\pm(0)\rangle$ replaced by

$$\begin{aligned}\langle\sigma_-(0)\rangle &\to \langle\sigma_+(t_0)\sigma_-(t_0)\rangle_{ss} = \frac{1}{2} - \frac{1/2}{2N+1},\\ \langle\sigma_+(0)\rangle &\to \langle\sigma_+(t_0)\sigma_+(t_0)\rangle_{ss} = 0.\end{aligned} \tag{14}$$

The fluorescence spectrum is given by $S(\omega) = \frac{\gamma}{2\pi}\mathrm{Re}\big\{\langle\widetilde{\sigma_+\sigma_-}\rangle(i\omega)\big\}$, the normalized form of which is the quantity plotted in Fig. 2.

We note that the discussion above essentially means that under the set of the adopted assumptions the quantum regression theorem can be applied despite the finite memory effects induced by the phase modulation.

In Fig. 3 we show how the phase modulation increases the sensitivity to squeezing. The graphs represent three possible reservoirs: squeezed phase modulated, squeezed unmodulated and white noise reservoir. One observes that the narrowing of the central peak is present even for the reduced values of M provided a strong phase modulation is applied.

We will now briefly discuss another example, the *linear phase modulation*

$$f(\omega) = T|\omega|, \tag{15}$$

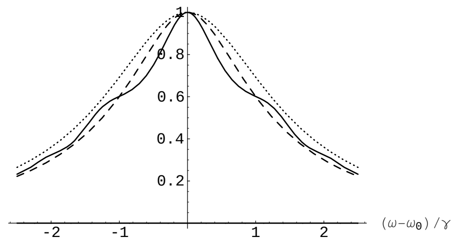

FIG. 3: Sensitivity of the fluorescence spectrum to the degree of squeezing. $T\gamma = 2, \mathcal{M} = 0.5$, (solid line), $T = 0, \mathcal{M} = 0.5$, (dashed) and $\mathcal{M} = 0$ (dotted), white noise reservoir. For all graphs $\mathcal{N} = 1$.

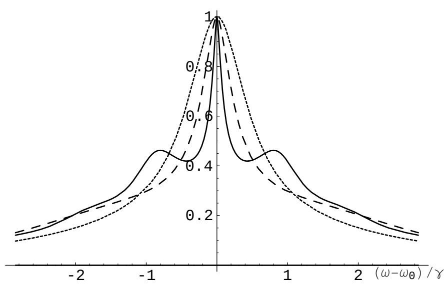

FIG. 4: Linear phase modulation (15) for $\mathcal{N} = 1, \mathcal{M} = 1$ and $T\gamma = 5$ (solid line), $T\gamma = 1$ (dashed) $T = 0$, (doted), the normal Markov case.

where T is a real positive parameter. In this case $\tilde{k}(s)$ is multivalued with $s = 0$ as a branch point. The pole structure is now more involved and should be discussed together with the branch cuts in the complex $Res < 0$ half plane. In particular the discussion following Eq. (9) should be modified. We will not discuss it here but rather present in Fig. 4 the quantity $\langle\tilde{\sigma}_-(s = i\omega)\rangle/\langle\tilde{\sigma}_-(s = i\omega_0)\rangle$ for this phase modulation. As discussed above this quantity represents the fluorescence spectrum. We observe features similar to those found in the quadratic modulation case and in particular the narrowing of the spectrum around the central frequency and the development of the side lobes. From an experimental point of view the advantages of the linear modulation is the relative ease of achieving it in practice.

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